Realizable spin models and entanglement dynamics in superconducting flux qubit systems

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Realizable spin models are investigated in a superconducting flux qubit system. By adjusting system parameters, it is shown that various artificial spin systems can be realized such as the quantum Ising model and the class of *XXZ* spin model in the same flux qubit system. The entanglement dynamics of the realizable systems, especially for the two-qubit system, is discussed by means of their concurrence and fidelity. It is found that an unentangled input state can evolve to be a maximally entangled output state periodically due to the exchange interactions induced by two-qubit flipping tunneling processes.

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INTRODUCTION

Much attention have been paid to superconducting qubit systems[1–3] as one of the promising candidates for quantum information processing and computing. The tunable superconducting devices have provided a variety of possibilities to realize quantum spin models that are not findable naturally. Recent experiments have shown that different types of exchange interactions are observable. Particularly, there have been demonstrated an Ising-type interaction in two charge qubits[4] and two flux qubits[5,6] and an *XY*-type interaction in two flux[7] and phase qubits.[8,9] Moreover, such realizations of artificial spin systems make it possible to observe entangled states of two qubits.[4,5,8,9] For a capacitively coupled two phase qubits, a recent experiment[9] shows that higher fidelity for the entanglement is exhibited in an excited level. The higher fidelity is caused by two-qubit tunneling processes[10] between two-qubit states, i.e., flipping both qubits. Such a two-qubit tunneling process contributes to (tunnel-type) exchange interactions between the two artificial spins.

In this paper, we will theoretically investigate a possible realization of quantum spin models in superconducting flux qubit systems by varying a system parameter. Especially, we use a phase coupling by introducing a connecting wire between the two-qubit loops (see Fig. 1)[11,12] because the phase coupling gives more controllable parameters than the inductive coupling for the manipulation of qubit states. For coupled charge qubits,[4] the Ising-type interaction has been generated, and for coupled phase qubits,[8,9] the tunnel-type exchange interactions have been demonstrated. However, in phase-coupled flux qubit systems, both the Ising- and tunnel-type interactions can be simultaneously generated.[10,13] Further, such an exchange interaction in flux qubit systems can be manipulated by controlling the values of system parameters. In this study, we numerically calculate both the tunnel- and Ising-type interaction strengths for three characteristic parameter regimes. As a result, we show that a flux qubit system can be an *artificial XXZ quantum spin system* by virtue of the present phase coupling scheme. In addition, to address about the time evolution of an input state for the two flux qubit system corresponding to quantum spin models, we introduce the concurrence and fidelity as a function of time as a measure of entanglement and evolution of the state. It turns out that an unentangled (entangled) input state evolves to be an entangled (unentangled) state periodically with a characteristic period of time.

MODEL

We first consider two coupled superconducting flux qubits in Fig. 1. The Hamiltonian describing the model is given by the sum of the charging and Josephson energies,

$$H(\{\dot{\varphi}_i,\dot{\varphi}',\varphi_i,\varphi'\}) = H_C(\{\dot{\varphi}_i,\dot{\varphi}'\}) + H_J(\{\varphi_i,\varphi'\}), \qquad (1)$$

where the phases across the Josephson junctions are φ_i and their time derivatives are $\dot{\varphi}_i$. The charging energy of Joseph-

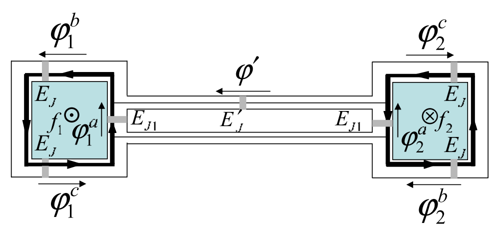

FIG. 1. (Color online) A two flux qubit system. The system is composed of two (left and right) qubit loops. In order to couple the two flux qubits, we use two connecting superconducting wires where the Josephson junction E_J' plays the important role for controlling the interaction between the two qubits since the two wires give the boundary condition as a function of phases $\{\varphi_1^a,\varphi_2^a,\varphi'\}$ from the fluxoid quantization along the closed path through the two connecting wires. By varying the amplitude of E_J', the two flux qubit system can be mapped into a quantum two-spin model. The state of each qubit loop is in a superposed state of which $|\downarrow\rangle$ and $|\uparrow\rangle$ represent the diamagnetic and paramagnetic current states, respectively. This schematic of the system show the state $|\downarrow\uparrow\rangle$ that is one of possible four states. Here, ⊙ and ⊗ denote the directions of the magnetic fields, $f_{1(2)}=\Phi_{1(2)}/\Phi_0$, in the qubit loops. E_{J1}, E_J, and E_J' are the Josephson coupling energies of the Josephson junctions in the qubit loops and the superconducting connecting wire, and φ's are phase differences across the Josephson junctions.

son junctions in the two-qubit loops and the connecting wire is given by

$$H_C = \frac{1}{2}\left(\frac{\Phi_0}{2\pi}\right)^2\left(\sum_{i=1}^{2}\sum_{\alpha\in\{a,b,c\}} C_i^{\alpha}\dot{\varphi}_i^{\alpha 2} + C'\dot{\varphi}'^2\right), \qquad (2)$$

where $C^{\alpha}(C')$ are the capacitance of the Josephson junctions in the qubit (connecting) loops. $\Phi_0 = h/2e$ is the unit flux quantum. The Josephson energy of the junctions is given by

$$H_J = \sum_{i=1}^{2}\sum_{\alpha\in\{a,b,c\}} 2E_{Ji}^{\alpha}\sin^2\frac{\varphi_i^{\alpha}}{2} + 2E_J'\sin^2\frac{\varphi'}{2}, \qquad (3)$$

where E_J's are the Josephson energy of junctions in the qubit and connecting loops.

The two current states of a flux qubit can be represented in terms of pseudospin language, i.e., two orthogonal states $|\uparrow\rangle$ and $|\downarrow\rangle$. Then, two-qubit systems can be represented in the basis $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$. In the low energy limit, the Hamiltonian of superconducting flux qubit systems can be represented in the tight-binding approximation. For two flux qubit systems, following Refs. 10 and 13, one can write the two-qubit matrix Hamiltonian in terms of qubit energy levels, single-qubit tunnelings, and two-qubit tunnelings,

$$H = \begin{pmatrix} E_{\uparrow\uparrow} & -t_1 & -t_1 & -t_2^a \\ -t_1 & E_{\uparrow\downarrow} & -t_2^b & -t_1 \\ -t_1 & -t_2^b & E_{\downarrow\uparrow} & -t_1 \\ -t_2^a & -t_1 & -t_1 & E_{\downarrow\downarrow} \end{pmatrix}, \qquad (4)$$

where E's are the energies of the Josephson junction loop for the two-qubit states. t_1 and t_2 are the single- and two-qubit tunnelings between the two states of two qubits, originating from the charging energies H_C. Single-qubit tunneling describes single-qubit flipping for the macroscopic quantum tunneling between the two states of the two-qubit states, for example, $|\uparrow\uparrow\rangle \Leftrightarrow |\downarrow\uparrow\rangle$. The two-qubit tunneling amplitudes, (i) t_2^a and (ii) t_2^b, describe the tunneling processes (i) $|\uparrow\uparrow\rangle \Leftrightarrow |\downarrow\downarrow\rangle$ in the parallel pseudospin states and (ii) $|\uparrow\downarrow\rangle \Leftrightarrow |\downarrow\uparrow\rangle$ in the antiparallel pseudospin states. The interaction between two qubits is controlled by adjusting the Josephson junction energy E_J' in the superconducting connecting wire[12] and the tunneling amplitudes are calculated numerically using both the Wentzel-Kramers-Brillouin approximation and the Fourier grid Hamiltonian method.[14,15]

In fact, the tunneling amplitudes and the low energy qubit energies are determined by the system parameters of the superconducting flux qubit system. Once the parameters are adjusted, generally, an artificial spin Hamiltonian is given in a form from Eq. (4),

$$H = \sum_{j\in\{1,2\}}\sum_{\alpha\in\{x,y,z\}} B_j^{\alpha}S_j^{\alpha} + \sum_{\alpha\in\{x,y,z\}} J_{\alpha}S_1^{\alpha}S_2^{\alpha}, \qquad (5)$$

where $B_j^x = -t_1$, $B_j^y = 0$, $B_1^z = (E_{\uparrow\uparrow} + E_{\uparrow\downarrow} - E_{\downarrow\uparrow} - E_{\downarrow\downarrow})/4$, $B_2^z = (E_{\uparrow\uparrow} - E_{\uparrow\downarrow} + E_{\downarrow\uparrow} - E_{\downarrow\downarrow})/4$, $J_x = -(t_2^a + t_2^b)/2$, $J_y = (t_2^a - t_2^b)/2$, $J_z = (E_{\uparrow\uparrow} - E_{\uparrow\downarrow} - E_{\downarrow\uparrow} + E_{\downarrow\downarrow})/4$, and S_j^{α}'s are the Pauli matrices. The single-qubit tunnelings play the role of a transverse magnetic field B_j^x, while the energy difference of two-qubit levels corresponds to the applied magnetic field B_j^z parallel to the z direction of spins. Note that the x and y components of the exchange interaction (J_x and J_y) are determined by the two-qubit tunnelings and the z component of the interaction (J_z) is the energy difference between the parallel spin state and the antiparallel spin state. Consequently, Eq. (5) shows that a kind of *XYZ* quantum spin model with magnetic fields can be realizable in the two flux qubit system.[13] Compared to the artificial spin interactions, a recent theoretical study shows that an unusual type of spin interactions, e.g., $S_1^zS_2^x$ and $S_1^xS_2^z$, can be realizable by applying a microwave in a superconducting qubit system.[16]

REALIZABLE ARTIFICIAL SPIN SYSTEMS

To manipulate two flux qubits in the model of Fig. 1, one can vary the energies of Josephson junctions in the connecting wires and the qubit loops. Varying the Josephson energies of the junctions determines the types of interactions between the two artificial spins. Three types of artificial spin models are found as follows.

Case I. For $E_J' = 0.0E_J$ and $E_{J1} = 0.7E_J$, a two-spin Hamiltonian can be constructed by the relations of $E_{\uparrow\uparrow} = E_{\uparrow\downarrow} = E_{\downarrow\uparrow} = E_{\downarrow\downarrow}$ and $t_2^a = t_2^b = t_2$. We calculated the numerical values of the macroscopic quantum tunnelings as $t_1 \approx 0.0075E_J$ and $t_2^{a(b)} \approx 0.00024E_J$.[10] Then, from Eq. (5), the two flux qubit system can be described by the corresponding spin Hamiltonian

$$H = JS_1^xS_2^x + B(S_1^x + S_2^x), \qquad (6)$$

where $B = -t_1$ and $J = -t_2$. The interaction J between the artificial spins is originated by the two-qubit tunneling t_2 between the two-qubit states. The Ising-type interaction is zero, $J_z = 0$. Rotating the coordinates, in which S_x becomes S_z, the spin model can be transformed to $\tilde{H} = JS_1^zS_2^z + B(S_1^z + S_2^z)$. The transformed Hamiltonian describes the quantum Ising model with an external magnetic field B.

Case II. When the Josephson energy E_J' increases from zero,[12] the Ising-type interaction J_z of the two flux qubits is generated. For $E_J' = 0.1E_J$ with $E_{J1} = 0.57E_J$, one finds the relations $E_{\uparrow\uparrow} = E_{\downarrow\downarrow} \neq E_{\downarrow\uparrow} = E_{\uparrow\downarrow}$ and $t_1 = t_2^b = 0$. Then, the corresponding spin Hamiltonian for two artificial spins can be written as

$$H = J(S_1^xS_2^x - S_1^yS_2^y) + J_zS_1^zS_2^z, \qquad (7)$$

where $J = -t_2^a/2$ and $J_z = (E_{\uparrow\uparrow} - E_{\uparrow\downarrow})/2$. The numerical values of the two-qubit tunneling amplitude and the energy difference between the two states are given by $t_2^a \approx 0.012E_J$ and $J_z \approx -0.1E_J$. It is shown that the tunnel-type exchange interaction J as well as the Ising-type exchange interaction J_z appears, while the single-qubit tunneling is suppressed. On the other hand, the inductively coupled flux qubits can provide either Ising-type[5,6] or tunnel-type exchange interactions.[7] For other types of superconducting qubits, only the tunnel-type (Ising-type) interaction can be generated in the phase (charge) qubits by introducing a coupling capacitance.

The Hamiltonian in Eq. (7) describes an anisotropic spin exchange interaction with $J < 0$ and $J_z < 0$. Interestingly, the

x and z components of the interaction are antiferromagnetic, while the y component is ferromagnetic. However, if one introduces a rotated coordinate for the first qubit such as $e^{i\sigma_1^y\pi/2}\sigma_1^z e^{-i\sigma_1^y\pi/2} = -\sigma_1^z$ and $e^{i\sigma_1^y\pi/2}\sigma_1^x e^{-i\sigma_1^y\pi/2} = -\sigma_1^x$, then one can transform the Hamiltonian $\widetilde{H} = e^{i\sigma_1^y\pi/2}He^{-i\sigma_1^y\pi/2}$. The transformed Hamiltonian describes an *XXZ* quantum spin model,

$$\widetilde{H} = -J(S_1^x S_2^x + S_1^y S_2^y) - J_z S_1^z S_2^z. \tag{8}$$

This implies that by studying the model of the spin Hamiltonian in Eq. (7), one can study the model of the spin Hamiltonian in Eq. (8) within the unitary transformation. Hence, the flux qubits for the parameters provide a class of general *XXZ* quantum spin models. Moreover, applying σ_1^x rotation instead of σ_1^y gives the opposite signs of the interactions in the two terms in Eq. (8).

Case III. For the intermediate values of $E_J' = 0.05E_J$ with $E_{J1} = 0.7E_J$, the single- and two-qubit tunneling amplitudes are $t_1 = 0.0024E_J$ and $t_2^a = 0.00024E_J$, respectively. In this case, the energy difference is given as $J_z = -0.05E_J$ and the two-qubit tunnelings is not negligible. Hence, another realization of quantum spin models is possible and its Hamiltonian can be written as

$$H = J(S_1^x S_2^x - S_1^y S_2^y) + J_z S_1^z S_2^z + B(S_1^x + S_2^x), \tag{9}$$

where $B = -t_1$, $J = -t_2^a/2$, and $J_z = (E_{\uparrow\uparrow} - E_{\uparrow\downarrow})/2$. In addition to the *XXZ* spin exchange interaction, it is shown that the single-qubit tunneling processes play a role of the magnetic fields. Then, this Hamiltonian describes the interacting two spins with magnetic fields.

TIME EVOLUTION OF QUANTUM STATES

Let us investigate the time evolution of a prepared quantum state in the realizable spin models. At $t=0$, any normalized pure state of two qubits (artificial spins) can be written as

$$|\psi(0)\rangle = a_0|\uparrow\uparrow\rangle + b_0|\uparrow\downarrow\rangle + c_0|\downarrow\uparrow\rangle + d_0|\downarrow\downarrow\rangle, \tag{10}$$

where a_0, b_0, c_0, and d_0 are the coefficients of the wave function. The entanglement can be quantified by the concurrence at time t,[17]

$$C(|\psi(t)\rangle) = 2|a(t)d(t) - b(t)c(t)|, \tag{11}$$

where $a(t)$, $b(t)$, $c(t)$, and $d(t)$ are the coefficients of the wavefunction at time t. The concurrence ranges from 0 (unentangled state) to 1 (a maximally entangled state). To help understanding the entanglement dynamics, one can define the overlap between the states at the initial time (input state) and at a given time t (output state) as the fidelity,

$$F(t) = |\langle\psi(t)|\psi(0)\rangle|. \tag{12}$$

If $F(T)=1$, the output quantum state is the same with the initial input state at $t=T$, i.e., the unentangled (entangled) initial state returns to the unentangled (entangled) state. Then, for time evolution of quantum states, entanglement dynamics can be understood from the concurrence and fidelity.

For the quantum Ising model of case I, the concurrence is given by

$$C(t) = [C_0 + C_1 \cos 4Jt]^{1/2}, \tag{13}$$

where $C_0 = [(a_0+d_0)^2 - (b_0+c_0)^2]^2/4 + [(a_0-d_0)^2 - (b_0-c_0)^2]^2/4$ and $C_1 = -[(a_0+d_0)^2 - (b_0+c_0)^2][(a_0-d_0)^2 - (b_0-c_0)^2]/2$. It should be noticed that the concurrence does not depend on the magnetic field $B = -t_1$, i.e., the single-qubit tunneling. Only the strength of the Ising interaction determines the dynamics of entanglement. This can be understood as follows. In the case of $J=0$, the spin Hamiltonian becomes $\widetilde{H} = B(S_1^z + S_2^z)$ below Eq. (6). The input state of Eq. (10) will precess around the magnetic field with $a(t) = a_0 e^{-2iBt}$, $b(t) = b_0$, $c(t) = c_0$, and $d(t) = d_0 e^{2iBt}$. Thus, the concurrence is constant in time. Even for $J \neq 0$, $[JS_1^z S_2^z, B(S_1^z + S_2^z)] = 0$ shows that the magnetic field does not affect on the time evolution of the state. As a result, the concurrence is an oscillating function with respect to the exchange interaction $J = -t_2$ with the characteristic period of time $T = \pi/2J$. At $t = 2m\pi/4J$ with an integer m, the concurrence reaches its initial value, $C = 2|a_0 d_0 - b_0 c_0|$, while at $t = (2m+1)\pi/4J$, $C = |a_0^2 - b_0^2 - c_0^2 + d_0^2|$.

The fidelity of this quantum spin system is given by

$$F(t) = \left[F_0 + \sum_{\sigma=\pm} F_1^\sigma \cos 2(B + \sigma J)t + F_2 \cos 4Bt\right]^{1/2}, \tag{14}$$

where $F_0 = 1 - [(a_0+d_0)^2 + (b_0+c_0)^2][(a_0-d_0)^2 + (b_0-c_0)^2]/2 - [(a_0+d_0)^2 - (b_0+c_0)^2]^2/8$, $F_1^+ = (a_0+b_0+c_0+d_0)^2[(a_0-d_0)^2 + (b_0-c_0)^2]/4$, $F_1^- = (a_0-b_0-c_0+d_0)^2[(a_0-d_0)^2 + (b_0-c_0)^2]/4$, and $F_2 = [(a_0+d_0)^2 - (b_0+c_0)^2]^2/8$.

For the *XXZ* spin model of case II, the concurrence is given by

$$C(t) = \left[C_0 + \sum_{\sigma=\pm} C_1^\sigma \cos 4(J + \sigma J_z)t + C_2 \cos 8Jt\right]^{1/2}, \tag{15}$$

where $C_0 = [(a_0+d_0)^4 + (a_0-d_0)^4]/4 + 4b_0^2 c_0^2$, $C_1^+ = -2(a_0+d_0)^2 b_0 c_0$, $C_1^- = 2(a_0-d_0)^2 b_0 c_0$, and $C_2 = -(a_0^2 - d_0^2)^2/2$ and the fidelity is

$$F(t) = \left[F_0 + \sum_{\sigma=\pm} F_1^\sigma \cos 2(J + \sigma J_z)t + F_2 \cos 4Jt\right]^{1/2}, \tag{16}$$

where $F_0 = 1 - 2(a_0^2 + d_0^2)(b_0^2 + c_0^2) - (a_0^2 - d_0^2)^2/2$, $F_1^+ = (a_0+d_0)^2(b_0^2 + c_0^2)$, $F_1^- = (a_0-d_0)^2(b_0^2 + c_0^2)$, and $F_2 = (a_0^2 - d_0^2)^2/2$. This shows that the concurrence and fidelity have a similar dynamic property. However, the fidelity has twice longer period than the concurrence. The reason is as follows: if we consider only the term $J_z S_1^z S_2^z$ in Eq. (8) for simplicity, the state, $|\psi(t)\rangle$, evolves as $a(t) = a_0 e^{-iJ_z t}$, $b(t) = b_0 e^{iJ_z t}$, $c(t) = c_0 e^{iJ_z t}$, and $d(t) = d_0 e^{-iJ_z t}$. Thus, the state $|\psi(t)\rangle$ and fidelity oscillate as $e^{2iJ_z t}$, but the concurrence, $C(t) = 2|a(t)d(t) - b(t)c(t)|$, as $e^{4iJ_z t}$.

In case III, the expressions of the concurrence and fidelity are too lengthy to display. One can find that the external field does not commute with the exchange interactions, i.e.,

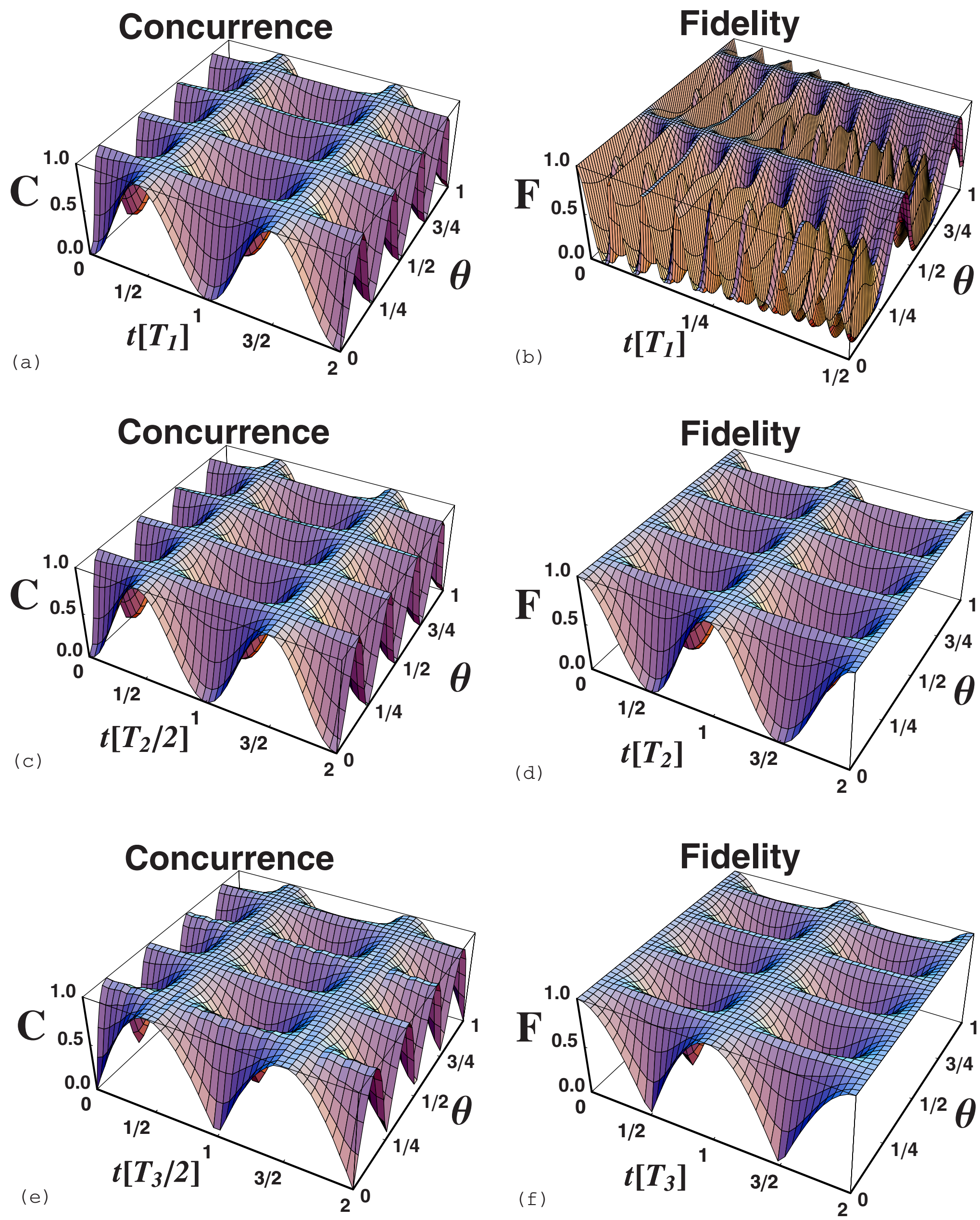

FIG. 2. (Color online) Time evolutions of the concurrence $C(t,\theta)$ and the fidelity $F(t,\theta)$ for the input state $|\Psi(0)\rangle = \cos 2\pi\theta|\uparrow\uparrow\rangle + \sin 2\pi\theta|\downarrow\downarrow\rangle$ at the coresonance point $\Phi_1=\Phi_2=0.5\Phi_0$ in the two superconducting flux qubit system. (a,b) Case I. For $E_J'=0.0E_J$ and $E_{J1}=0.7E_J$, the two flux qubit system corresponds to the spin Hamiltonian $H=JS_1^xS_2^x+B(S_1^x+S_2^x)$ with the exchange interaction $J=-t_2^a$ and the magnetic field $B=-t_1$. The single- and two-qubit tunneling amplitudes are given by $t_1=0.0075E_J$ and $t_2=0.000\,24E_J$. The characteristic period of time is $T_1=\pi/2J$ for the concurrence. (c,d) Case II. For $E_J'=0.6E_J$ and $E_{J1}=0.7E_J$, the two flux qubit system maps into the spin Hamiltonian $H=J(S_1^xS_2^x-S_1^yS_2^y)+J_zS_1^zS_2^z$ with the exchange interactions $J=-t_2^a/2$ and $J_z=(E_{\uparrow\uparrow}-E_{\uparrow\downarrow})/2$. The two-qubit tunneling amplitude is $t_2^a=0.000\,24E_J$ and the energy difference between the two states is $J_z=-0.425\,045E_J$. The period of time is $T_2/2$ with $T_2=\pi/2J$. The fidelity has twice the period of the concurrence. (e,f) Case III. For $E_J'=0.05E_J$ and $E_{J1}=0.7E_J$, the two flux qubit system is described by the spin Hamiltonian $H=J\,(S_1^xS_2^x-S_1^yS_2^y)+J_zS_1^zS_2^z+B(S_1^x+S_2^x)$ with the magnetic field $B=-t_1$ and the exchange interactions $J=-t_2^a/2$ and $J_z=(E_{\uparrow\uparrow}-E_{\uparrow\downarrow})/2$. The single- and two-qubit tunneling amplitudes are $t_1=0.0024E_J$ and $t_2^a=0.000\,24E_J$ and the energy difference becomes $J_z=-0.05E_J$. The characteristic period of the time evolutions is $T_3=0.68\pi/2J$. The fidelity has twice the period of the concurrence.

$[-JS_1^yS_2^y+J_zS_1^zS_2^z, B(S_1^x+S_2^x)]\neq 0$. Then, the shape of concurrence deforms depending on the external field B.

In Fig. 2, we plot the concurrences and fidelities as a function of time t and the initial state parameter θ to give the comparison of entanglement dynamics between the different spin models for the same initial state $|\Psi(0)\rangle=\cos 2\pi\theta|\uparrow\uparrow\rangle + \sin 2\pi\theta|\downarrow\downarrow\rangle$. Explicitly, the different values of system parameters controlling the two flux qubits are given in the captions of the figures. For the time evolution of the initial state, it is shown that the unentangled (entangled) state can become an entangled (unentangled) state even though the specifications of the superconducting devices are different to each other. In Fig. 2(a), for example, the state becomes maximally entangled with $C=1$ at $t=(2m+1)\pi/4J$. Further, for the special case that $\theta=(2n+1)/8$ with integer n, the concurrence in Eq. (13) results in $C(t)=1$: the state is always maximally entangled, while the fidelity is oscillating in time.

Although we have discussed about the two coupled qubit case so far, it is possible to couple many qubits to form a one-dimensional spin chain. By performing a coordinate transformation similar to that in the above of Eq. (8) for a sublattice, the coupled qubit array will be described by the *XXZ* spin chain Hamiltonian. Then, we can obtain and simulate an artificial *XXZ* spin chain model.

SUMMARY

A two superconducting flux qubit system has been considered to investigate a possible realization of quantum spin models. The phase-coupled flux qubits can provide both the Ising- and tunnel-type interactions between two qubits, resulting in the class of *XXZ* quantum spin model. The artificial quantum Ising and *XXZ* spin models were demonstrated by varying controllable system parameters. Further, we discussed the entanglement dynamics of the artificial spin models in the specific parameter values of the two superconducting flux qubit system. It was found that a certain class of input unentangled (entangled) state can become a maximally entangled (unentangled) state irrespective of the specifications of the superconducting devices. Such a maximally entangled state should be observable experimentally.

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